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**CURRENT  $s$  - QUARK MASS CORRECTIONS TO THE FORM  
FACTORS OF  $D$  - MESON SEMILEPTONIC DECAYS**

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**Abstract**

The infinite mass effective theory, when a heavy quark mass tends to infinity, and Chiral perturbation theory at the quark level, based on the extended Nambu – Jona – Lasinio model with linear realization of chiral  $U(3) \times U(3)$  symmetry, are applied to the calculation of current  $s$  – quark mass corrections to the form factors of the  $D \rightarrow \bar{K} e^+ \nu_e$  and  $D \rightarrow \bar{K}^* e^+ \nu_e$  decays. These corrections turn out to be quite significant, of the order of 7 – 20%. The theoretical results are compared with experimental data.

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# 1 Introduction

In our recent publications [1,2] we calculated in the chiral limit the form factors of the semileptonic  $D \rightarrow \bar{K}^* e^+ \nu_e$  and  $D \rightarrow \bar{K} e^+ \nu_e$  decays. For the description of  $D$ -mesons we applied the infinite mass effective theory (IMET) [3,4], when the  $c$ -quark mass  $M_c$  tends to infinity, i.e.  $M_c \rightarrow \infty$ . In IMET the long – distance physics we describe within Chiral perturbation theory at the quark level (CHPT) $_q$  [5], based on the extended Nambu – Jona – Lasinio (ENJL) model with linear realization of chiral  $U(3) \times U(3)$  symmetry [6].

In this paper we apply IMET and (CHPT) $_q$  to the calculation of the fine structure of the form factors of the  $D \rightarrow \bar{K} e^+ \nu_e$  and  $D \rightarrow \bar{K}^* e^+ \nu_e$  decays at the first order in current  $s$  – quark mass expansion. Within IMET and (CHPT) $_q$  the first order current – quark – mass corrections to the mass spectra of charmed pseudoscalar and vector mesons and charmed pseudoscalar – meson leptonic constants have been calculated in [7]. The amplitude of the  $D \rightarrow h e^+ \nu_e$  decay can be determined as follows

$$M(D \rightarrow h e^+ \nu_e) = -\frac{G_F}{\sqrt{2}} V_{cs}^* \langle h(Q) | \bar{s}(0) \gamma_\mu (1 - \gamma^5) c(0) | D(p) \rangle \ell^\mu, \quad (1)$$

where  $h = \bar{K}$  or  $\bar{K}^*$ ,  $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$  is Fermi weak constant,  $|V_{cs}| = 0.975$  is the CKM – mixing matrix element,  $s(0)$  and  $c(0)$  are the  $s$  – and  $c$  – current quark fields with  $N$  colour degrees of freedom each, and  $\ell^\mu = \bar{u}(k_{\nu_e}) \gamma^\mu (1 - \gamma^5) v(k_{e^+})$  is the weak leptonic current.

We shall seek the hadronic matrix element

$$M_\mu(D \rightarrow h) = \langle h(Q) | \bar{s}(0) \gamma_\mu (1 - \gamma^5) c(0) | D(p) \rangle \quad (2)$$

in the form of an expansion in powers of the current  $s$  – quark mass upto first order terms

$$M_\mu(D \rightarrow h) = M_\mu^{(0)}(D \rightarrow h) + M_\mu^{(1)}(D \rightarrow h). \quad (3)$$

Here we have denoted

$$M_\mu^{(0)}(D \rightarrow h) = \langle h(Q) | \bar{s}(0) \gamma_\mu (1 - \gamma^5) c(0) | D(p) \rangle_{\text{ch.l.}} \quad (4)$$

$$\begin{aligned} M_\mu^{(1)}(D \rightarrow h) = \\ = -i m_{0s} \int d^4 x \langle h(Q) | T(\bar{s}(x) s(x) \bar{s}(0) \gamma_\mu (1 - \gamma^5) c(0)) | D(p) \rangle_{\text{ch.l.}}. \end{aligned} \quad (5)$$

The matrix element  $M_\mu^{(0)}(D \rightarrow h)$  describes the  $D \rightarrow h$  transition calculated in the chiral limit (ch.l.) while  $M_\mu^{(1)}(D \rightarrow h)$  is the first order correction in the current  $s$  – quark mass expansion. The matrix elements  $M_\mu^{(0)}(D \rightarrow h)$  for  $h = \bar{K}^*$  and  $\bar{K}$  have been calculated in [1,2]. In this paper we shall calculate  $M_\mu^{(1)}(D \rightarrow h)$ .

In accordance to the procedure expounded in [1,2,7] we reduce (5) to the expression

$$\begin{aligned} M_\mu^{(1)}(D \rightarrow h) = g_D m_{0s} i \int d^4 x, \int_{-\infty}^{\infty} dz_0 \theta(-z_0) \times \\ \times \langle h(Q) | T(\bar{s}(x) s(x) s(0) \gamma_\mu \left( \frac{1 + \gamma^0}{2} \right) \gamma^5 q(z_0, \vec{0})) | 0 \rangle_{\text{ch.l.}} \end{aligned} \quad (6)$$

obtained in leading order in the large  $N$  and  $M_c$  expansion,  $q = u$  or  $d$  for  $D^0$  or  $D^+$ , respectively. The coupling constant  $g_D$  has been calculated in [8]

$$g_D = \frac{2\sqrt{2}\pi}{\sqrt{N}} \left( \frac{M_D^2}{M_c \bar{v}'} \right)^{1/2}, \quad (7)$$

where  $\bar{v}' = 4\Lambda = 2.66$  GeV and  $\Lambda$  is the cut-off in 3-dimensional Euclidean momentum space.  $\Lambda$  is connected to the scale of spontaneous breaking of chiral symmetry (SBCS)  $\Lambda_\chi$  by the relation  $\Lambda = \Lambda_\chi/\sqrt{2} = 0.66$  GeV at  $\Lambda_\chi = 0.94$  GeV [5].

The r.h.s. of (6) involves only the light-quark fields. Therefore for the evaluation of (6) we can apply  $(\text{CHPT})_q$  [5,7]. Since the leading order of the r.h.s. of (6) in current-quark-mass expansion is fixed by the factor  $m_{0s}$ , so the matrix element  $\langle h(Q) | T(\dots) | 0 \rangle$  has to be calculated in the chiral limit (ch.l.).

By applying the formulas of quark conversion (Ivanov [5]) we can present the matrix element  $M_\mu^{(1)}(D \rightarrow h)$  in terms of constituent-quark-loop diagrams [7]. The momentum representation of these diagrams reads

$$\begin{aligned} M_\mu^{(1)}(D \rightarrow h) &= i m_{0s} g_D g_h \frac{\bar{v}}{4m} \left( -\frac{N}{16\pi^2} \right) \int \frac{d^4k}{\pi^2 i} \text{tr} \left[ \frac{1}{m - \hat{k}} \Gamma_h \times \right. \\ &\times \left. \frac{1}{m - \hat{Q} - \hat{k}} \frac{1}{m - \hat{Q} - \hat{k}} \gamma_\mu (1 - \gamma^5) \left( \frac{1 + \hat{v}}{2} \right) \gamma^5 \frac{1}{k \cdot v + i0} \right]. \end{aligned} \quad (8)$$

The appearance of the factor  $\bar{v}/4m$  is due to the contribution of the diagram with the light scalar  $\sigma_s$ -meson exchange [8]. Here  $\bar{v} = -\langle 0 | \bar{q}q | 0 \rangle / F_0^2 = 1.92$  GeV,  $F_0 = 0.092$  GeV and  $m = 0.33$  GeV are the PCAC constant of light pseudoscalar mesons and the constituent quark mass calculated in the chiral limit [5]. The coupling constants  $g_h$  describe the interaction between light constituent quarks and light mesons  $\bar{K}$  and  $\bar{K}^*$ , that is  $g_{\bar{K}} = 2\pi/\sqrt{N}$  and  $g_{\bar{K}^*} = \pi\sqrt{6}/\sqrt{N}$  [7,8] such that  $g_{\bar{K}^*}/g_{\bar{K}} = \sqrt{3/2}$  [5].  $\Gamma_h$  is either  $\Gamma_{\bar{K}} = i\gamma^5$  or  $\Gamma_{\bar{K}^*} = \gamma_\nu e^{*\nu}(Q)$  depending on whether  $h = \bar{K}$  or  $h = \bar{K}^*$ . Now we can proceed to the calculation of the current  $s$ -quark mass corrections to the form factors.

## 2 The $D \rightarrow \bar{K} e^+ \nu_e$ decay

For the  $D \rightarrow \bar{K}$  the matrix element  $M_\mu^{(1)}(D \rightarrow \bar{K})$  reads

$$\begin{aligned} M_\mu^{(1)}(D \rightarrow \bar{K}) &= i m_{0s} g_D \frac{2\pi}{\sqrt{N}} \frac{\bar{v}}{4m} \left( -\frac{N}{16\pi^2} \right) \int \frac{d^4k}{\pi^2 i} \text{tr} \left[ \frac{1}{m - \hat{k}} i\gamma^5 \times \right. \\ &\times \left. \frac{1}{m - \hat{Q} - \hat{k}} \frac{1}{m - \hat{Q} - \hat{k}} \gamma_\mu (1 - \gamma^5) \left( \frac{1 + \hat{v}}{2} \right) \gamma^5 \frac{1}{k \cdot v + i0} \right] = \\ &= m_{0s} g_D \frac{\sqrt{N}}{4\pi} \int \frac{d^4k}{\pi^2 i} \frac{k_\mu}{[m^2 - k^2 - i0][m^2 - (k + Q)^2 - i0]} \frac{1}{k \cdot v + i0} + \\ &+ \dots \end{aligned} \quad (9)$$

Following [9,10] we have kept only divergent contributions. The dots denote the contributions of convergent integrals. The integration over  $k$  gives [1]

$$\begin{aligned}
& \int \frac{d^4 k}{\pi^2 i} \frac{k_\mu}{[m^2 - k^2 - i0][m^2 - (k + Q)^2 - i0]} \frac{1}{k \cdot v + i0} = \\
& = v_\mu 2 \ell n \left( 1 + \frac{\bar{v}'}{4 Q_0} \right) + Q_\mu \frac{2}{Q_0} \left[ 1 - \ell n \left( 1 + \frac{\bar{v}'}{4 Q_0} \right) \right]
\end{aligned} \tag{10}$$

where  $Q_0 = (M_D^2 - q^2)/2 M_D$  is the energy of the massless  $K$  - meson in the rest frame of the  $D$  - meson. The appearance of the  $q^2$  - dependence is due to the  $q^2$  - dependence of  $Q_0$ . The matrix element  $M_\mu^{(1)}(D \rightarrow \bar{K})$  can be expressed in terms of two form factors

$$M_\mu^{(1)}(D \rightarrow \bar{K}) = f_+^{(1)}(q^2)(p + Q)_\mu + f_-^{(1)}(q^2)(p - Q)_\mu \tag{11}$$

where

$$\begin{aligned}
f_+^{(1)}(q^2) &= \frac{m_{0s}}{M_*} \frac{\bar{v}}{4m} \frac{M_D^2}{M_D^2 - q^2} \left[ 1 - \frac{M_D^2 + q^2}{2 M_D^2} \ell n \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right) \right], \\
f_-^{(1)}(q^2) &= -\frac{m_{0s}}{M_*} \frac{\bar{v}}{4m} \frac{M_D^2}{M_D^2 - q^2} \left[ 1 - \frac{3 M_D^2 - q^2}{2 M_D^2} \ell n \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right) \right].
\end{aligned} \tag{12}$$

Here we have denoted  $2 M_* = \sqrt{2 M_D \bar{v}'}$ . It should be stressed that the formulae (12) are valid in the physical region only, i.e.  $0 \leq q^2 \leq (M_D - m_K)^2$ . At  $q^2 = 0$  the form factors  $f_+^{(1)}(0)$  and  $f_-^{(1)}(0)$  read

$$\begin{aligned}
f_+^{(1)}(0) &= \frac{m_{0s}}{M_*} \frac{\bar{v}}{4m} \left[ 1 - \frac{1}{2} \ell n \left( 1 + \frac{M_*^2}{M_D^2} \right) \right] = 0.09, \\
f_-^{(1)}(0) &= -\frac{m_{0s}}{M_*} \frac{\bar{v}}{4m} \left[ 1 - \frac{3}{2} \ell n \left( 1 + \frac{M_*^2}{M_D^2} \right) \right] = -0.02.
\end{aligned} \tag{13}$$

In the chiral limit the quantity  $f_+(0)$  has been calculated in [8] (see also [2])

$$f_+^{(0)}(0) = \frac{1}{\sqrt{2}} \left( \frac{\bar{v}'}{2 M_c} \right)^{1/2} = 0.6. \tag{14}$$

The numerical value is estimated at the equality  $M_c = M_D = 1.86$  GeV accepted in our approach [9]. By adding the current  $s$  - quark mass correction (13) we get the total value of  $f_+(0)$

$$f_+(0) = f_+^{(0)}(0) + f_+^{(1)}(0) = 0.69 \tag{15}$$

which is good compared with the experimental data  $|f_+(0)|_{\text{exp}} = 0.7 \pm 0.1$  [11]. Our result  $f_+(0) = 0.69$  agrees well with the theoretical prediction by Dominguez and Paver [12] obtained within the QCD sum rule approach. We find the current  $s$ -quark mass correction to be about 15%.

### 3 The $D \rightarrow \bar{K}^* e^+ \nu_e$ decay

The matrix element  $M_\mu^{(1)}(D \rightarrow \bar{K}^*)$  can be expressed in terms of four form factors [1]

$$\begin{aligned} M_\mu^{(1)}(D \rightarrow \bar{K}^*) &= i a_1^{(1)}(q^2) e_\mu^*(Q^2) - i a_2^{(1)}(q^2) (e^*(Q) \cdot p) (p + Q)_\mu - \\ &- i a_3^{(1)}(q^2) (e^*(Q) \cdot p) (p - Q)_\mu - \\ &- 2 b^{(1)}(q^2) \varepsilon_{\mu\nu\alpha\beta} e^{t\nu}(Q) p^\alpha Q^\beta, (\varepsilon^{0123} = 1). \end{aligned} \quad (16)$$

In order to obtain the form factors  $a_i^{(1)}(q^2)$  ( $i = 1, 2, 3$ ) and  $b^{(1)}(q^2)$  we have to calculate the following momentum integral

$$\begin{aligned} \mathcal{M}_{\mu\nu} &= \int \frac{d^4 k}{\pi^2 i} \text{tr} \left[ \frac{1}{m - \hat{k}} \gamma_\nu \frac{1}{m - \hat{Q} - \hat{k}} \frac{1}{m - \hat{Q} - \hat{k}} \times \right. \\ &\quad \left. \times \gamma_\mu (1 - \gamma^5) \left( \frac{1 + \hat{v}}{2} \right) \gamma^5 \frac{1}{k \cdot v + i0} \right]. \end{aligned} \quad (17)$$

By keeping only divergent contributions [9,10] and using the integrals given in the Appendix of [1], we get

$$\begin{aligned} \mathcal{M}_{\mu\nu} &= -4 \ell n \left( \frac{\bar{v}'}{4m} \right) g_{\mu\nu} + \frac{8}{M_D^2 - q^2} \left[ 1 - \ell n \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right) \right] Q_\mu p_\nu - \\ &- \frac{8i}{M_D^2 - q^2} \left[ 1 - \ell n \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right) \right] \varepsilon_{\mu\nu\alpha\beta} p^\alpha Q^\beta. \end{aligned} \quad (18)$$

The appearance of the  $q^2$  - dependence is due to the quantity  $Q_0 = (M_D^2 - q^2)/2M_D$ , being the energy of the massless  $\bar{K}^*$  - meson in the rest frame of the  $D$  - meson. The neglect of the  $\bar{K}^*$  - meson mass in the r.h.s. of (17) is in accordance with the prescription of  $(\text{CHPT})_q$  which incorporates the Vector Dominance approach [5,13], admitting the smooth dependence of low - energy hadronic matrix elements on the masses of low -lying vector mesons ( $\rho, \omega, \varphi, K^*$ ) [1,13,14].

By using (18), one can calculate the following chiral corrections to the form factors of the  $D \rightarrow \bar{K}^*$  transition

$$\begin{aligned} a_1^{(1)}(q^2) &= \frac{\sqrt{3}}{2} \frac{m_{0s}}{M_*} \frac{\bar{v}}{4m} M_D \ell n \left( \frac{\bar{v}'}{4m} \right) \\ a_2^{(1)}(q^2) &= -a_3^{(1)}(q^2) = b^{(1)}(q^2) \\ b^{(1)}(q^2) &= \frac{\sqrt{3}}{2} \frac{m_{0s}}{M_*} \frac{\bar{v}}{4m} \frac{M_D}{M_D^2 - q^2} \left[ 1 - \ell n \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right) \right]. \end{aligned} \quad (19)$$

In the chiral limit the form factors of the  $D \rightarrow \bar{K}^*$  transition have been calculated in [1]

$$\begin{aligned}
a_1^{(0)}(q^2) &= \sqrt{\frac{3}{8}} M_* \\
a_2^{(0)}(q^2) &= \sqrt{\frac{3}{8}} \frac{M_*}{M_D^2} \left[ \frac{q^2}{M_D^2 - q^2} + \right. \\
&\quad \left. + \frac{M_D^2 - q^2}{M_*^2} \left( 1 - \frac{2m M_D}{M_D^2 - q^2} \right) \ell n \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right) \right], \\
a_3^{(0)}(q^2) &= -\sqrt{\frac{3}{8}} \frac{M_*}{M_D^2} \left[ \frac{2M_D^2 - q^2}{M_D^2 - q^2} - \right. \\
&\quad \left. - \frac{M_D^2 - q^2}{M_*^2} \left( 1 - \frac{2m M_D}{M_D^2 - q^2} \right) \ell n \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right) \right], \\
b^{(0)}(q^2) &= \sqrt{\frac{3}{8}} \frac{1}{M_*} \ell n \left( 1 + \frac{M_*^2}{M_D^2 - q^2} \right).
\end{aligned} \tag{20}$$

Here we have used the relation (14). The numerical values of the form factors at  $q^2 = 0$  read

$$\begin{aligned}
a_1(0) &= a_1^{(0)}(0) + a_1^{(1)}(0) = 0.96 + 0.14 = 1.10 \text{ (GeV)}, \\
a_2(0) &= a_2^{(0)}(0) + a_2^{(1)}(0) = 0.14 + 0.03 = 0.17 \text{ (GeV)}^{-1}, \\
a_3(0) &= a_3^{(0)}(0) + a_3^{(1)}(0) = -0.42 - 0.03 = -0.45 \text{ (GeV)}^{-1}, \\
b(0) &= b^{(0)}(0) + b^{(1)}(0) = 0.21 + 0.03 = 0.24 \text{ (GeV)}^{-1}.
\end{aligned} \tag{21}$$

One sees that the first order current  $s$ -quark mass mass corrections are between 7 and 20%. The form factors  $a_i(q^2)$  ( $i = 1, 2, 3$ ) and  $b(q^2)$  are connected with the standard form factors  $A_i(q^2)$  ( $i = 1, 2, 3$ ) and  $V(q^2)$  via the relations [1]

$$\begin{aligned}
A_1(q^2) &= \frac{1}{M_D + M_{\bar{K}^*}} a_1(q^2)|_{q^2=0} = 0.40 \\
A_2(q^2) &= (M_D + M_{\bar{K}^*}) a_2(q^2)|_{q^2=0} = 0.47 \\
A_3(q^2) &= (M_D + M_{\bar{K}^*}) a_3(q^2)|_{q^2=0} = -1.24 \\
V(q^2) &= (M_D + M_{\bar{K}^*}) b(q^2)|_{q^2=0} = 0.66,
\end{aligned} \tag{22}$$

where  $M_{\bar{K}^*} = 0.89 \text{ GeV}$  is the mass of the  $\bar{K}^*$  - meson [11]. The theoretical values compare reasonably with recent experimental data [15]

$$\begin{aligned}
A_1(0)_{exp} &= 0.46 \pm 0.05 \pm 0.05, \\
A_2(0)_{exp} &= 0.38 \pm_{0.12}^{0.11} \pm 0.07, \\
V(0)_{exp} &= 0.92 \pm_{0.18}^{0.19} \pm 0.12.
\end{aligned} \tag{23}$$

These numerical results obtained by taking into account the first order current  $s$  - quark mass corrections confirm the results found in [1]. It is because in [1] we expressed the form factors of the  $D \rightarrow \bar{K}^*$  transition in terms of the form factor of the  $D \rightarrow \bar{K}$  transition  $f_+(0)$ . There for the numerical estimate we used the value  $f_+(0) = 0.7$ , which we obtained in present paper only at the first order in current  $s$  - quark mass

expansion (15). Recall that in the chiral limit we have  $f_+(0) = 0.6$ . This overlap of results underscores the self-consistency of the current  $s$ -quark mass corrections to the form factors of the transitions  $D \rightarrow \bar{K}^*$  and  $D \rightarrow \bar{K}$  calculated within IMET and  $(\text{CHPT})_q$ .

## 4 Conclusion

We have applied IMET and  $(\text{CHPT})_q$  for the computation of the current  $s$ -quark mass corrections to the form factors of the semileptonic decays of the non-strange charmed  $D$ -mesons,  $D \rightarrow \bar{K} e^+ \nu_e$  and  $D \rightarrow \bar{K}^* e^+ \nu_e$ . We have obtained non-zero contributions for the first order corrections in current  $s$ -quark mass expansion to the form factors of the  $D \rightarrow \bar{K} e^+ \nu_e$  decays. This result contradicts the Ademollo-Gatto theorem for the form factors of the semileptonic decays of  $K$ -mesons [16]. Within  $(\text{CHPT})_q$  the Ademollo-Gatto theorem has been analyzed in [17]. The observed contradiction can be explained as an effect of IMET. Indeed IMET is based on the infinite limit  $M_c \rightarrow \infty$  which violates chiral  $SU(4) \times SU(4)$  symmetry, a necessary condition for the validity of the Ademollo-Gatto theorem for the  $D \rightarrow \bar{K} e^+ \nu_e$  decays. The current  $s$ -quark mass corrections to the form factors of the  $D \rightarrow \bar{K}^* e^+ \nu_e$  decays are consistent with the corrections calculated for the form factors of the  $D \rightarrow \bar{K} e^+ \nu_e$  decays.

Note that we have kept to the first order corrections in current  $s$ -quark mass expansion calculated at the tree-meson level. Of course, the one-meson-loop corrections can be taken into account too. The consistent procedure for meson-loop chiral corrections within  $(\text{CHPT})_q$  has been developed in Ref.[5]). This procedure can also be applied to charmed meson physics.

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